

磁学部分第三次答案

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5.2 解答

磁矩是磁化强度与体积的乘积:

$$m = VM = \frac{1}{4}\pi D^2 LM = \frac{1}{4}\pi \times 0.01^2 \times 0.03 \times 1,200 = 2.8 \times 10^{-3} \text{ A} \cdot \text{m}^2.$$

5.4 解答

(1) 原子的个数为

$$N = \frac{m}{M} N_A = \frac{\frac{1}{4}\pi \rho D^2 L}{M} N_A.$$

电子的个数为

$$N_e = 2N = \frac{\pi \rho D^2 L}{2M} N_A = \frac{\pi \times 7.8 \times 1.0^2 \times 12}{2 \times 55.85} \times 6.02 \times 10^{23} = 1.58 \times 10^{24}.$$

(2) 电子的磁矩大小是玻尔磁矩

$$m_B = \frac{eh}{4\pi m_e} = 9.27 \times 10^{-24} \text{ A} \cdot \text{m}^2,$$

其中 $h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s}^{-1}$ 是普朗克常数。因此, 总磁矩为

$$\mu = N_e m_B = 1.58 \times 10^{24} \times 9.27 \times 10^{-24} = 14.6 \text{ A} \cdot \text{m}^2.$$

(3) 对应电流为

$$I = \frac{\mu}{\frac{1}{4}\pi D^2} = \frac{14.6}{\frac{1}{4}\pi \times 0.01^2} = 1.86 \times 10^5 \text{ A}.$$

面电流密度为

$$J = \frac{I}{L} = \frac{1.86 \times 10^5}{0.12} = 1.55 \times 10^6 \text{ A} \cdot \text{m}^{-1}.$$

(4) 产生的磁场为

$$B = \frac{\mu_0 I}{L} = \mu_0 J = 4\pi \times 10^{-7} \times 1.55 \times 10^6 = 1.95 \text{ T}.$$

5.5 解答

(1) 根据安培环路定律, 磁场强度为

$$H = \frac{NI}{P} = \frac{400 \times 20}{0.40} = 2.0 \times 10^4 \text{ A} \cdot \text{m}^{-1}.$$

(2) 磁化强度为

$$M = \frac{B}{\mu_0} - H = \frac{1.0}{4\pi \times 10^{-7}} - 2.0 \times 10^4 = 7.8 \times 10^5 \text{ A} \cdot \text{m}^{-1}.$$

(3) 磁化率为

$$\chi = \frac{M}{H} = \frac{7.8 \times 10^5}{2.0 \times 10^4} = 39.$$

(4) 磁化面电流为 $J = M = 7.8 \times 10^5 \text{ A} \cdot \text{m}^{-1}$ 。相对磁导率为 $\mu_r = 1 + \chi = 40$ 。

5.6 解答

只需将 4.9 题的 Q 替代为 $4\pi\epsilon_0RV$ 即可。因此

$$m = \frac{1}{3}\omega(4\pi\epsilon_0RV)R^2 = \frac{4}{3}\pi\omega\epsilon_0R^3V.$$

5.8 解答

根据图表可得, 当磁场为 $B = 1.2 \text{ T}$ 时, 磁场强度约为 $H = 220 \text{ A} \cdot \text{m}^{-1}$ 。如果将铁芯抽出, 为了使磁场依然为 $B = 1.2 \text{ T}$, 那么此时, 磁场强度为

$$H' = \frac{B}{\mu_0} = \frac{1.2}{4\pi \times 10^{-7}} = 9.55 \times 10^5 \text{ A} \cdot \text{m}^{-1}.$$

因为磁场强度与通入电流成正比, 因此, 此时导线通入电流为

$$I' = \frac{H'}{H}I = \frac{9.55 \times 10^5}{220} \times 6.0 = 2.60 \times 10^4 \text{ A}.$$

5.10 解答

假设在空气一侧与软铁一侧, 磁感应强度与磁场强度的垂直、水平分量分别为

$$B_1^\perp, B_1^\parallel; B_2^\perp, B_2^\parallel; H_1^\perp, H_1^\parallel; H_2^\perp, H_2^\parallel.$$

根据边界条件可得

$$B_1^\perp = B_2^\perp,$$

$$H_1^\parallel = H_2^\parallel.$$

而磁感应强度与磁场强度的关系取决于相对磁导率的大小, 即

$$B_1^\perp = \mu_{r1}\mu_0H_1^\perp,$$

$$B_1^{\parallel} = \mu_{r1}\mu_0 H_1^{\parallel},$$

$$B_2^{\perp} = \mu_{r2}\mu_0 H_2^{\perp},$$

$$B_2^{\parallel} = \mu_{r2}\mu_0 H_2^{\parallel}.$$

因此

$$\frac{\tan \theta_1}{\tan \theta_2} = \frac{|B_1^{\parallel}|}{|B_1^{\perp}|} / \frac{|B_2^{\parallel}|}{|B_2^{\perp}|} = \frac{|B_1^{\parallel}|}{|B_1^{\perp}|} / \frac{|B_2^{\parallel}|}{|B_2^{\perp}|} = \frac{\mu_{r1}\mu_0 |H_1^{\parallel}|}{\mu_{r1}\mu_0 |H_1^{\perp}|} / \frac{\mu_{r2}\mu_0 |H_2^{\parallel}|}{\mu_{r2}\mu_0 |H_2^{\perp}|} = \frac{\mu_{r1}}{\mu_{r2}}.$$

代入 $\theta_1 = 85^\circ$, $\mu_{r1} = 1$, $\mu_{r2} = 7,000$ 可得 $\theta_2 = 5.6'$, 其中 1 分为 1/60 度。

5.14 解答

(1) 根据安培环路定律可得, 平板之间的磁场强度为

$$H = i_0.$$

因此, 上半部分与下半部分的磁感应强度分别为

$$B_1 = \mu_{r1}\mu_0 H = \mu_{r1}\mu_0 i_0,$$

$$B_2 = \mu_{r2}\mu_0 H = \mu_{r2}\mu_0 i_0.$$

(2) 上半部分与下半部分的磁化强度分别为

$$M_1 = \frac{B_1}{\mu_0} - H = (\mu_{r1} - 1)i_0,$$

$$M_2 = \frac{B_2}{\mu_0} - H = (\mu_{r2} - 1)i_0.$$

因此, 上表面、交界面与下表面的磁化电流面密度分别为

$$i_{m1} = 0 - M_1 = -(\mu_{r1} - 1)i_0,$$

$$i_{m2} = M_1 - M_2 = (\mu_{r1} - \mu_{r2})i_0,$$

$$i_{m3} = M_2 - 0 = (\mu_{r2} - 1)i_0.$$

5.15 解答

很显然，磁场强度方向是竖直向上的，记为

$$\mathbf{H} = (0, H(x, y, z), 0).$$

因此，磁场强度的旋度为

$$\operatorname{curl} \mathbf{H} = (-\partial_z H, 0, \partial_x H).$$

而电流密度分布为

$$\mathbf{J} = \begin{cases} (0, 0, j), & -d/2 < x < d/2; \\ (0, 0, -j), & 3d/2 < x < 5d/2; \\ 0, & \text{otherwise.} \end{cases}$$

根据 $\operatorname{curl} \mathbf{H} = \mathbf{J}$ 可得

$$\partial_z H \equiv 0,$$

$$\partial_x H = \begin{cases} j, & -d/2 < x < d/2; \\ -j, & 3d/2 < x < 5d/2; \\ 0, & \text{otherwise.} \end{cases}$$

根据边界条件 $H|_{x \leq -d/2} \equiv 0, H|_{x \geq 5d/2} \equiv 0$ 可得

$$H(x, y, z) = \begin{cases} jx + jd/2, & -d/2 < x < d/2; \\ jd, & d/2 < x < 3d/2; \\ 5jd/2 - jx, & 3d/2 < x < 5d/2; \\ 0, & \text{otherwise.} \end{cases}$$

磁感应强度分布为

$$B(x, y, z) = \mu_r(x, y, z)\mu_0 H(x, y, z) = \begin{cases} \mu_{r1}\mu_0(jx + jd/2), & -d/2 < x < d/2; \\ \mu_{r1}\mu_0 jd, & d/2 < x < 3d/2; \\ \mu_{r2}\mu_0(5jd/2 - jx), & 3d/2 < x < 5d/2; \\ 0, & \text{otherwise.} \end{cases}$$

5.16 解答

很显然，磁场强度方向是垂直于纸面的，记为

$$\mathbf{H} = (0, 0, H(x, y, z)).$$

因此，磁场强度的旋度为

$$\operatorname{curl} \mathbf{H} = (\partial_y H, -\partial_x H, 0).$$

而电流密度分布为

$$\mathbf{J} = \begin{cases} (0, j, 0), & -b/2 < x < b/2; \\ 0, & \text{otherwise.} \end{cases}$$

根据 $\text{curl } \mathbf{H} = \mathbf{J}$ 可得

$$\partial_y H \equiv 0,$$

$$\partial_x H = \begin{cases} -j, & -b/2 < x < b/2; \\ 0, & \text{otherwise.} \end{cases}$$

根据边界条件 $H|_{x=0} \equiv 0$ 可得

$$H(x, y, z) = \begin{cases} jb/2, & x \leq -b/2; \\ -jx, & -b/2 < x < b/2; \\ -jb/2, & x \geq b/2. \end{cases}$$

磁感应强度分布为

$$B(x, y, z) = \mu_r(x, y, z)\mu_0 H(x, y, z) = \begin{cases} \mu_{r1}\mu_0 jb/2, & x \leq -b/2; \\ -\mu_0 jx, & -b/2 < x < b/2; \\ -\mu_{r2}\mu_0 jb/2, & x \geq b/2. \end{cases}$$