

电磁学 C 期中复习课讲义

张竞一

2025 年 10 月 26 日

1 一些注意事项

1. 电势和场强的关系:

$$\mathbf{E} = E_x \mathbf{e}_x + E_y \mathbf{e}_y + E_z \mathbf{e}_z = -\frac{\partial U}{\partial x} \mathbf{e}_x - \frac{\partial U}{\partial y} \mathbf{e}_y - \frac{\partial U}{\partial z} \mathbf{e}_z = -\nabla U$$

2. 对 \mathbb{R}^2 中 $dxdy$ 换元为极坐标系中 $drd\theta$ 时需要记住 Jacobi 行列式带入后的表达式:

$$dxdy = r dr d\theta$$

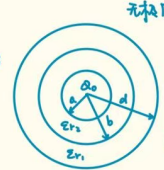
3. 导体板电荷分布中最外侧为 $\frac{1}{2} \Sigma Q$, 相邻导体板面的电荷互为相反数。

4. 只有线性极化的电介质才满足 $D = \epsilon_0 \epsilon_r E$, 非线性极化不能用这个公式。

5. 需要熟记平行板电容器, 球形电容器, 圆柱形电容器等常见电容器的电容公式。

2 一些常见题型

1. 同心圆柱



无相反自由电荷 (有相反电荷前合起来自由电荷分布不变) $E_0 = \begin{cases} 0 & r < a \\ \frac{Q_0}{4\pi\epsilon_0 r^2} & r > a \end{cases}$

则 $E = \begin{cases} 0 & r < a \\ \frac{1}{\epsilon_1} \frac{Q_0}{4\pi r^2} & a < r < b \\ \frac{1}{\epsilon_2} \frac{Q_0}{4\pi r^2} & b < r < d \\ \frac{Q_0}{4\pi\epsilon_0 r^2} & r > d \end{cases}$ $D = \begin{cases} 0 & r < a \\ \frac{Q_0}{4\pi r^2} & r > a \end{cases}$

由于 $D = \epsilon_0 \epsilon_r E \Rightarrow p = D - \epsilon_0 E = \epsilon_0 (\epsilon_r - 1) E = \frac{\epsilon_r - 1}{\epsilon_r} D$

则 $p = \begin{cases} 0 & r < a \\ \frac{\epsilon_2 - 1}{\epsilon_2} \frac{Q_0}{4\pi r^2} & a < r < b \\ \frac{\epsilon_1 - 1}{\epsilon_1} \frac{Q_0}{4\pi r^2} & b < r < d \\ 0 & (\epsilon_r = 1) \quad r > d \end{cases}$


$(E = \frac{1}{\epsilon_r} E_0 \text{ 或用 } E = \frac{1}{\epsilon_r \epsilon_0} D \text{ 来算})$

对于 σ' (面极化电荷密度), σ_0 (面自由电荷密度)

$\sigma' = -\hat{n} \cdot (\mathbf{P}^{(1)} - \mathbf{P}^{(2)})$ $\sigma_0 = \hat{n} \cdot (\mathbf{D}^{(1)} - \mathbf{D}^{(2)})$, \hat{n} 取外法向

$\sigma_0 = \epsilon_0 \hat{n} \cdot (\mathbf{E}^{(1)} - \mathbf{E}^{(2)})$ (总电荷)

2. 圆筒



$a < r < b$ 时 $E?$

$\oint \mathbf{D} \cdot d\mathbf{s} = \epsilon_1 \epsilon_0 E \cdot 2\pi r^2 + \epsilon_2 \epsilon_0 E \cdot 2\pi r^2 = \frac{Q_0}{\epsilon_0}$

$\Rightarrow E = \frac{Q_0}{(\epsilon_1 + \epsilon_2) \epsilon_0 2\pi r^2}$

用 $D = \epsilon_0 \epsilon_r E \rightarrow D_1 = \frac{\epsilon_1}{\epsilon_1 + \epsilon_2} \frac{Q_0}{2\pi r^2}$ $P_1 = (\epsilon_1 - 1) \epsilon_0 E = P_1 = \frac{\epsilon_1 - 1}{\epsilon_1 + \epsilon_2} \frac{Q_0}{2\pi r^2}$

$D_2 = \frac{\epsilon_2}{\epsilon_1 + \epsilon_2} \frac{Q_0}{2\pi r^2}$ $P_2 = \frac{\epsilon_2 - 1}{\epsilon_1 + \epsilon_2} \frac{Q_0}{2\pi r^2}$

σ', σ_0 :

① 在 ϵ_1, ϵ_2 界面上 $\sigma' = \sigma_0 = 0$ (\hat{n} 与 \mathbf{D} 和 \mathbf{P} 均垂直)

② 在 $r=a$ 界面上: $\sigma' = -\hat{n} \cdot (\mathbf{P}^{(1)} - \mathbf{P}^{(2)}) = \begin{cases} -P_1(a) & (1\text{ 侧}) \\ -P_2(a) & (2\text{ 侧}) \end{cases}$ (注意通入下压 (界面上用总压))

$\sigma_0 = \hat{n} \cdot (\mathbf{D}^{(1)} - \mathbf{D}^{(2)}) = \begin{cases} D_1(a) & (1\text{ 侧}) \\ D_2(a) & (2\text{ 侧}) \end{cases}$

$\oint \mathbf{E} \cdot d\mathbf{s} = E \cdot 4\pi r^2 \neq \frac{Q_0}{\epsilon_0}$ \checkmark Q 为自由电荷 Q_0 加上极化电荷 Q'

$\oint \mathbf{D} \cdot d\mathbf{s} \neq \frac{Q_0}{\epsilon_0}$ \checkmark D 用 Gauss 定理用下 Q 为自由电荷量

D 不对称, 与 ϵ_r 有关

$\epsilon_r = 1$

③ 在 $r=b$ 界面上

$\mathbf{P}^{(1)} = 0$ $\mathbf{D}^{(1)} =$

$\mathbf{P}^{(2)} = P_2(b) / P_1(b)$

$\mathbf{D}^{(2)} = D_2(b) / D_1(b)$

静电学

$W = \frac{1}{2} \iiint \rho(\vec{r}) U(\vec{r}) dV$

1. 体电荷分布: $W_e = ?$

$W_e = \frac{1}{2} \iiint \rho U dV = \int \frac{1}{2} \rho U + r^2 dr$

$U(r) = \int_r^{\infty} E dr = \int_r^R \frac{\rho r'}{3\epsilon_0} dr' + \int_R^{\infty} \frac{\rho R^3}{5\epsilon_0 r'^2} dr' = \frac{\rho}{6\epsilon_0} (3R^2 + r^2)$

$W_e = \frac{1}{2} \rho \int_0^R 4\pi r^2 \cdot \frac{\rho}{6\epsilon_0} (3R^2 + r^2) dr = \frac{4\pi \rho^2 R^5}{15\epsilon_0}$

2. 球壳: $W_e = ?$

求两个球壳 W_{e1}, W_{e2}

$W_{e1} = \frac{1}{2} Q_1 U_1 + \frac{1}{2} Q_2 U_1 + \frac{1}{2} Q_1 U_2$

$U_1 = \frac{Q_1 + Q_2 + Q_3}{4\pi\epsilon_0 d}$

$U_2 = \frac{1}{4\pi\epsilon_0} (\frac{Q_1 + Q_2}{b} + \frac{Q_3}{d})$

$U_3 = \frac{1}{4\pi\epsilon_0} (\frac{Q_1}{a} + \frac{Q_2}{b} + \frac{Q_3}{d})$

$W_{e12} = \frac{1}{2} \cdot \frac{Q_1^2}{C_1 + C_2} \quad C_1 = \frac{4\pi\epsilon_0 ab}{b-a} \quad C_2 = \frac{4\pi\epsilon_0 bd}{d-b}$

$W_{e12} = \frac{1}{2} \cdot \frac{(Q_1 + Q_2 + Q_3)^2}{4\pi\epsilon_0 d}$

$W_{e3} = W_{e12} + W_{e11}$

对于球壳: $U = \begin{cases} \frac{Q}{4\pi\epsilon_0 r} & r > R \\ \frac{Q}{4\pi\epsilon_0 R} & r < R \end{cases}$ ($E=0$ 于壳内部与壳外电荷相等)

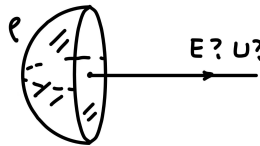
3 补充习题内容

下面的习题是助教在电磁学 B 期中考试前徐春凯老师给出的题型，能够帮助大家过一遍重要知识点，虽然是电磁学 B 要求内容，但是并不超纲，相信大家掌握后能对电磁学中电这一部分有一个更深刻的理解。

一. 电场与电势

电场和电势的部分是我们最开始学习的内容，一般来说考核方式比较简单，大多数情况是计算对称情形下电场和电势的表达式。

一个半球，体电荷密度为 ρ ，半径为 R ，求半球的大圆面那一端圆心向外垂直于平面的射线上电场强度 E 和电势 U 的大小



写出原表达式！考试中如果结果错误可以按照步骤给分，一定把步骤写全。

$$dE = \frac{dq}{4\pi\epsilon_0 l^2}, \quad dq = \rho dV, \quad dV = r^2 \sin\theta dr d\theta d\phi$$

$$l^2 = r^2 + x^2 + 2rx\cos\theta, \quad E = E_x, \quad dE_x = dE \cdot \cos\alpha, \quad \cos\alpha = \frac{x + r\cos\theta}{l}$$

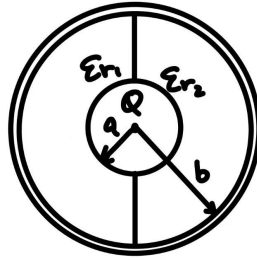
则

$$dE_x = \frac{\rho}{4\pi\epsilon_0} \cdot \frac{(x + r\cos\theta)r^2 \sin\theta dr d\theta d\phi}{(r^2 + x^2 + 2rx\cos\theta)^{\frac{3}{2}}}, \quad E = E_x = \int_0^R dr \int_0^{\frac{\pi}{2}} d\theta \int_0^{2\pi} \frac{\rho}{4\pi\epsilon_0} \cdot \frac{(x + r\cos\theta)r^2 \sin\theta d\phi}{(r^2 + x^2 + 2rx\cos\theta)^{\frac{3}{2}}}$$

$$dU = \frac{\rho}{4\pi\epsilon_0} \cdot \frac{r^2 \sin\theta dr d\theta d\phi}{(r^2 + x^2 + 2rx\cos\theta)^{\frac{1}{2}}}, \quad U = \int_0^R dr \int_0^{\frac{\pi}{2}} d\theta \int_0^{2\pi} \frac{\rho}{4\pi\epsilon_0} \cdot \frac{r^2 \sin\theta d\phi}{(r^2 + x^2 + 2rx\cos\theta)^{\frac{1}{2}}}$$

(这里只是给出解题方法，这个表达式有些复杂，感兴趣的同学可以继续向下计算)

二. 电介质球综合



(1) E, D, P?

有公式 $\iint \mathbf{D} ds = Q$, 则

$$\iint \mathbf{D} ds = 2\pi r^2 (\epsilon_0 \epsilon_{r1} E + \epsilon_0 \epsilon_{r2} E) = Q$$

于是有

$$E = \frac{Q}{2\pi r^2 \epsilon_0 (\epsilon_{r1} + \epsilon_{r2})}$$

由 $D = \epsilon_0 \epsilon_r E$, 有 $D_1 = \epsilon_0 \epsilon_{r1} E$, $D_2 = \epsilon_0 \epsilon_{r2} E$

由 $P = \epsilon_0 (\epsilon_r - 1) E$, 有 $P_1 = \epsilon_0 (\epsilon_{r1} - 1) E$, $P_2 = \epsilon_0 (\epsilon_{r2} - 1) E$

(2) σ', σ_0 ?

$$\sigma' = -\hat{n}(\mathbf{P}^{(2)} - \mathbf{P}^{(1)}), \sigma_0 = \hat{n}(\mathbf{D}^{(2)} - \mathbf{D}^{(1)})$$

在介质 1,2 的界面上 $\sigma' = 0, \sigma_0 = 0$

①当 $r = a$ 时,

$$\mathbf{P}^{(1)} = \mathbf{0}, \quad \mathbf{P}^{(2)} = \mathbf{P}; \quad \mathbf{D}^{(1)} = \mathbf{0}, \quad \mathbf{D}^{(2)} = \mathbf{D}$$

于是

$$\sigma'_1(a) = -\mathbf{P}_1(a), \quad \sigma'_2(a) = -\mathbf{P}_2(a); \quad \sigma_{01}(a) = \mathbf{D}_1(a), \quad \sigma_{02}(a) = \mathbf{D}_2(a)$$

②当 $r = b$ 时,

$$\mathbf{P}^{(1)} = \mathbf{P}, \quad \mathbf{P}^{(2)} = \mathbf{0}; \quad \mathbf{D}^{(1)} = \mathbf{D}, \quad \mathbf{D}^{(2)} = \mathbf{0}$$

于是

$$\sigma'_1(b) = \mathbf{P}_1(b), \quad \sigma'_2(b) = \mathbf{P}_2(b); \quad \sigma_{01}(b) = -\mathbf{D}_1(b), \quad \sigma_{02}(b) = -\mathbf{D}_2(b)$$

(3) 电势差 U, C ?

$$U = \int_a^b E dr = \frac{Q}{2\pi \epsilon_0 (\epsilon_{r1} + \epsilon_{r2})} \left(\frac{1}{a} - \frac{1}{b} \right), \quad C = \frac{Q}{U} = \frac{2\pi \epsilon_0 (\epsilon_{r1} + \epsilon_{r2})}{\frac{1}{a} - \frac{1}{b}}$$

(4) 储能 W_{e1}, W_{e2} ?

有

$$w_{e1} = \frac{1}{2} E_1 D_1 = \frac{1}{2} \epsilon_0 \epsilon_{r1} \frac{Q^2}{4\pi^2 \epsilon_0^2 (\epsilon_{r1} + \epsilon_{r2})^2 r^4} = \frac{\epsilon_{r1} Q^2}{8\pi^2 \epsilon_0 (\epsilon_{r1} + \epsilon_{r2})^2 r^4}$$

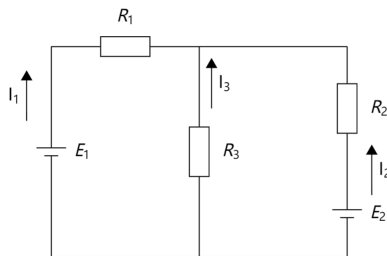
则

$$W_{e_1} = \int w_{e_1} dV = \int_a^b 2\pi r^2 w_{e_1} dr = \frac{\epsilon_{r_1} Q^2}{4\pi^2 \epsilon_0 (\epsilon_{r_1} + \epsilon_{r_2})^2} \left(\frac{1}{a} - \frac{1}{b} \right)$$

W_{e_2} 同理可求。

三.Kirchhoff 定律

求 I_1, I_2, I_3



$$\begin{cases} I_1 + I_2 + I_3 = 0 \\ E_1 = I_1 R_1 - I_3 R_3 \\ -E_2 = I_3 R_3 - I_2 R_2 \end{cases}$$

可以由此方程组解出 I 的值。