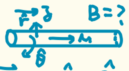
面磁化电流: $\oint_L \vec{M}\cdot d\vec{l} = \sum_{L内} I'$, 界面上: $\vec{i}' = \vec{n}\times(\vec{M}^{(2)}-\vec{M}^{(1)})$, $\vec{M} = nI_a\vec{S}_a$

$\vec{i}_t = \vec{n}\times\frac{1}{\mu_0}(\vec{B}^{(2)}-\vec{B}^{(1)})$ $\vec{i}_0 = \vec{n}\times(\vec{H}^{(2)}-\vec{H}^{(1)})$

例:  B=? 均匀磁化棒 M已知

$\vec{M} = M\hat{z}$ $\hat{S}_{底} = \pm\hat{z}$ 底面: $\vec{i}' = \vec{M}\times\hat{n} = 0$ 端: $B = \frac{1}{2}\mu_0 i' = \frac{1}{2}\mu_0 M$ (半无限长)

侧面: $\vec{i}' = \vec{M}\times\hat{r} = M\hat{\theta}$ 中: $B = \mu_0 i' = \mu_0 M$ (无限长)

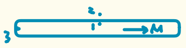
已知$\vec{M}$,有: $\vec{M}\to\vec{i}'\to\vec{B}$

均匀磁化球内 $B = \frac{2\mu_0 M}{3}$

$\oint \vec{H}\cdot d\vec{l} = \sum I_0$. $\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$ 磁化率: χ_m. $\vec{M} = \chi_m\vec{H}$

$\vec{B} = \mu_0(\vec{H}+\vec{M})$ $\vec{B} = \mu_0(1+\chi_m)\vec{H} = \mu_0\mu_r\vec{H}$

$\mu_r = 1+\chi_m$ 相对磁导率

例:  $B_z = 0$.

$\vec{B} = \mu_0(\vec{H}+\vec{M})$ $\vec{B} = \mu_0\mu_r\vec{H}$

$\vec{M} = (\mu_r-1)\vec{H}$

$H_1 = \frac{B_1}{\mu_0\mu_r} = \frac{M}{\mu_r}$ $B_1 = \mu_0 M$

$H_2 = \frac{B_2}{\mu_0} = 0$ $B_2 = 0$ $B_3 = \frac{1}{2}\mu_0 M$

类比:

电动势 ε — 磁动势 $\varepsilon_m = NI_0$

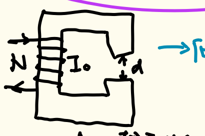
电流 I — 磁通量 $\Phi_B = \iint \vec{B}\cdot d\vec{S}$

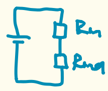
电阻 R — 磁阻 $R_m = \frac{l}{\mu_0\mu_r S}$

通 $\Phi = \iint \vec{B}\cdot d\vec{S}$

动 $\varepsilon_m = nI_0$

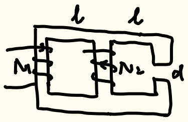
阻 $R_m = \frac{l}{\mu_0\mu S}$

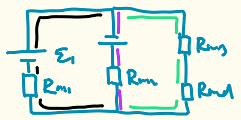
例题: 

→回路图为 

$R_m = \frac{4l-d}{\mu_0\mu_r S}$ $R_{md} = \frac{d}{\mu_0 S}$ ($\mu_r = 1$)

$\varepsilon_m = NI_0 = \underbrace{BS}_{\Phi}\cdot(R_m+R_{md})$ B可得



回路图为 

$R_{m1} = \frac{3l}{\mu_0\mu_r S}$ $R_{m2} = \frac{l}{\mu_0\mu_r S}$ $R_{m3} = \frac{3l-d}{\mu_0\mu_r S}$ $R_{md} = \frac{d}{\mu_0 S}$

基本方程: $\begin{cases}\nabla\cdot\vec{B} = 0\\ \nabla\times\vec{H} = \vec{j}_0\\ \vec{B} = \mu_0\mu_r\vec{H}\end{cases}$ 介质分区连续:

① 各向同性均匀介质充满磁场空间.

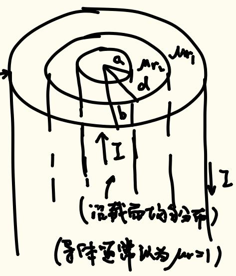
$\oint \vec{B}\cdot d\vec{l} = \begin{cases}\mu_0 I \text{ (总电流)}\\ \mu_0\mu_r\oint\vec{H}\cdot d\vec{l} = \mu_0\mu_r I_0 \text{ (传导电流)}\end{cases}$

$\Rightarrow B = \mu_r B_0$

② 不同介质,界面平行于B.

$\oint \vec{H}\cdot d\vec{l} = I_0$

$\oint \vec{B}_0\cdot d\vec{l} = \mu_0 I$ $\Rightarrow$ 若 $\oiint \vec{H}\cdot d\vec{S} = 0$. 有 $H = \frac{B_0}{\mu_0}$ (此时成立)

薄壳 

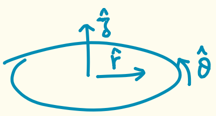
$\vec{B}, \vec{H}, \vec{i}_0, \vec{i}'$?

$\oint \vec{H}\cdot d\vec{l} = I_{0内}$

$H\cdot 2\pi r = \begin{cases}0 & r>b\\ I & a<r<b\\ \frac{r^2}{a^2}I & r<a\end{cases} \Rightarrow H = \begin{cases}0 & r>b\\ \frac{I}{2\pi r} & a<r<b\\ \frac{r}{2\pi a^2}I & r<a\end{cases}$

$M = (\mu_r-1)H = \begin{cases}0 & r>b\\ (\mu_{r1}-1)\frac{I}{2\pi r} & d<r<b\\ (\mu_{r2}-1)\frac{I}{2\pi r} & a<r<d\\ 0 & r<a\end{cases}$

$B = \mu_0\mu_r H = \begin{cases}0 & r>b\\ \frac{I}{2\pi\mu_0\mu_{r1}r} & d<r<b\\ \frac{I}{2\pi\mu_0\mu_{r2}r} & a<r<d\\ \frac{I}{2\pi a^2\mu_0}r & r<a\end{cases}$



$\hat{r}\times\hat{\theta} = \hat{z}$

$\vec{i}_0 = \hat{n}\times(\vec{H}^{(2)}-\vec{H}^{(1)})$ $(\hat{n} = \hat{r})$

$\vec{i}' = \hat{n}\times(\vec{M}^{(2)}-\vec{M}^{(1)})$

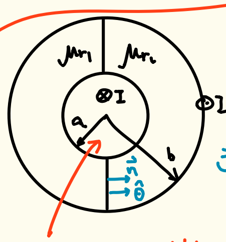
① $r=a$ $\vec{H}^{(2)} = \frac{I}{2\pi a}\hat{\theta}$, $\vec{H}^{(1)} = \frac{I}{2\pi a}\hat{\theta}$ $\vec{i}_0 = 0$.

$\vec{M}^{(2)} = (\mu_{r2}-1)\frac{I}{2\pi r}$ $\vec{M}^{(1)} = 0$. $\vec{i}' = (\mu_{r2}-1)\frac{I}{2\pi r}\cdot\hat{z}$

② $r=d$ 同理

③ $r=b$. $\vec{H}^{(2)} = 0$. $\vec{H}^{(1)} = \frac{I}{2\pi b}$ $\vec{i}_0 = -\frac{I}{2\pi b}\hat{z}$, $\vec{i}'$ 同理

③ 不同介质，界面垂直于B. 只适用面电流



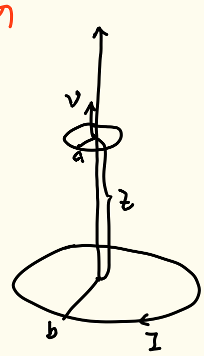
$$\oint \vec H\, d\vec l = \pi r\left(\frac{B}{\mu_0\mu_{r1}}+\frac{B}{\mu_0\mu_{r2}}\right)=I \Rightarrow B=\frac{\mu_{r1}\mu_{r2}\mu_0 I}{(\mu_{r1}+\mu_{r2})\pi r}$$

于是 $\hat n\times\hat\theta=0$（界面无电流

B连续，不会突变.

一定不是体心的！！！ $B_内$一定为0！

电流线圈：



$z\gg b\gg a$.

① a中电动势 ε_a?

② a受力 F_a?

① $\varepsilon_A=-\frac{d\Phi}{dt}$ ← 一定带原始公式.

$B_a=\frac{\mu_0 I}{2}\cdot\frac{b^2}{(b^2+z^2)^{3/2}}\approx\frac{\mu_0 I}{2}\cdot\frac{b^2}{z^3}$

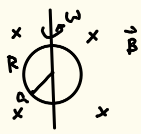
$\Phi_a=B_a\cdot\pi a^2$

$\Rightarrow \varepsilon_a=-\frac{d\Phi}{dt}=-\frac{d\Phi}{dz}\cdot\frac{dz}{dt}=\frac{3}{2}\frac{\mu_0 I a^2 b^2}{z^4}\cdot v$

②. $I_a=\frac{\varepsilon_a}{R}$. $m_a=I_a\cdot\pi a^2$

$F_a=m_a\cdot\frac{\partial}{\partial z}B_a$

对一段导体：$\varepsilon=\int_a^b \vec v\times\vec B\, d\vec l$. $u_{ab}=-\varepsilon_{ab}$ 对开路成立.

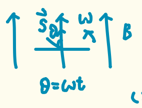


t=0时与B垂直.

(1) ε? I?

(2) 保持匀ω旋转的外力矩?

(1) $\Phi=B\cdot\pi a^2\cos\omega t \Rightarrow \varepsilon=-\frac{d\Phi}{dt}=\pi a^2B\omega\sin\omega t$



$I=\frac{\varepsilon}{R}=\frac{\pi a^2B\omega}{R}\sin\omega t$

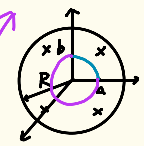
(2) $\mu=I\cdot\pi a^2=\frac{\pi^2a^4B\omega}{R}\sin\omega t$ （μ与S方向一致）（μ为磁矩）

通用的 $M=\mu B\sin\omega t$

$M_外=M=\frac{\pi^2a^4B^2\omega}{R}\sin^2\omega t$ 方向相反.

或：$P_\varepsilon=\frac{\varepsilon^2}{R}=M_外\cdot\omega$ （洛力不做功，电源功率与外力功率相同）

E涡



$\frac{dB}{dt}=k$ ① E的分布？

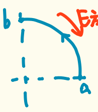
② 求 U_{ab} 1> 导线第一象限连接（导线为圆弧）

2> -- 2.3.4 ----

3> 无导线

①. $\oint\vec E\, d\vec l=-\frac{d\Phi}{dt}$ 即 $E\cdot2\pi r=\begin{cases}r<R & -\frac{d}{dt}(\pi r^2B)\\ r>R & -\frac{d}{dt}(\pi R^2B)\end{cases}$ 于是 $E=\begin{cases}r<R & -\frac{r}{2}\frac{dB}{dt}=-\frac{kr}{2}\\ r>R & -\frac{R^2}{2r}\frac{dB}{dt}=-\frac{kR^2}{2r}\end{cases}$

② 1>



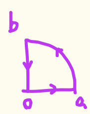
$U_{ab}=-\varepsilon_{a\to b}$

$\varepsilon_{a\to b}=\int_a^b\vec E_旋\cdot d\vec l=-E\cdot\frac14\cdot2\pi r_0=+\frac14\pi r_0^2k$，则 $U_{ab}=-\varepsilon_{a\to b}=-\frac14\pi r_0^2k$.

积分方向相反

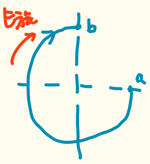
正负与方向是重要的.

若加 oa. ob.



由于 oa, ob与E垂直 于是此闭合回路电动势就是ab的电动势

2>.

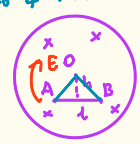


$U_{ab}=-\varepsilon_{a\to b}$

$\varepsilon_{a\to b}=\int_a^b\vec E_旋\, d\vec l=\frac34\cdot2\pi r_0\cdot-\frac{r_0}{2}k=-\frac34\pi k r_0^2$. 于是有 $U_{ab}=-\varepsilon_{ab}=\frac34\pi k r_0^2$

3>. $U_{ab}=0$ 两点间无电动势！电动势一定有路径.

如下题：



$\frac{dB}{dt}=k$. $U_{AB}=$?

$\varepsilon_{A\to B}=\varepsilon_{ABOA}=-\frac{d\Phi}{dt}=-\frac{lh}{2}\left(\frac{dB}{dt}\right)=\frac12 lhk$ 环流方向！！

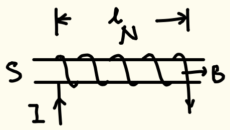
$U_{AB}=-\varepsilon_{A\to B}=-\frac12 Lhk$

$U_{AB}=-\varepsilon_{A\to B}=\varepsilon_{B\to A}$

$\varepsilon_{B\to A}=\varepsilon_{BAOB}=-\frac{d\Phi'}{dt}=\frac{lh}{2}\cdot\left(-\frac{dB}{dt}\right)=-\frac{lhk}{2}$

或：$\Phi_{OBAO}=\frac12 lhB$

$\varepsilon_{OBAO}=-\frac12 lhk=\varepsilon_{B\to A}=-\varepsilon_{A\to B}=U_{AB}$.

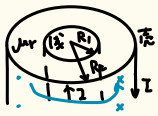


$B=\mu_0\mu_r\frac{N}{l}I$

$\mu_0\mu_r n I$

↑ 指单元函数

$\Phi=NBS=\mu_0\mu_r\frac{N^2}{l^2}ISl=\mu_0\mu_r n^2 VI \Rightarrow$ 自感 $I=\mu_0\mu_r n^2 V$



单位长度自感 L ?

① 高频近似 (表面流过!)

② 稳恒电流

$w_m=\frac{1}{2}\vec{B}\cdot\vec{H}$

$H=\begin{cases}\frac{r}{2\pi R_1^2}I & r<R_1\\ \frac{I}{2\pi r} & R_1<r<R_2\\ 0 & r>R_2\end{cases}$ $\Rightarrow w_m=\begin{cases}\frac{\mu_0 r^2}{8\pi^2R_1^4}I^2 & r<R_1\\ \frac{\mu_0\mu_r I^2}{8\pi^2 r^2} & R_1<r<R_2\end{cases}$

单长: $W_m=\left(\int_0^{R_1}+\int_{R_1}^{R_2}\right)w_m dr$ ✗

$\int_{1,2}$ 为 $w_m\cdot 2\pi r dr$

$=\frac{\mu_0 I^2}{16\pi}+\frac{\mu_0\mu_r I^2}{4\pi}\ln\frac{R_2}{R_1}$

$\frac{\mu_0 r^4}{16\pi R_1^4}I^2$ $L=\frac{2W}{I^2}$

① $\oint\vec{H}d\vec{l}=I_{内}=\begin{cases}0 & r<R_1\\ I & R_1<r<R_2\\ 0 & r>R_2\end{cases}$ 于是 $H=\begin{cases}\frac{I}{2\pi r} & R_1<r<R_2\\ 0 & \text{others}\end{cases}$. $B=\begin{cases}\frac{\mu_0\mu_r I}{2\pi r} & R_1<r<R_2\\ 0 & \text{others}\end{cases}$

$\oint\vec{H}d\vec{l}=2\pi rH$

dr, 1 $\Rightarrow ds=dr\times 1=dr$ 于是 $\Phi=\int_{R_1}^{R_2}\frac{\mu_0\mu_r I}{2\pi r}dr=\frac{\mu_r\mu_0}{2\pi}I\ln\frac{R_2}{R_1}\Rightarrow$ 自感 $L=\frac{\Phi}{I}=\frac{\mu_0\mu_r}{2\pi}\ln\frac{R_2}{R_1}$

不是 $2\pi r dr$! 看B方向!

② $\oint\vec{H}d\vec{l}=I_{内}=\begin{cases}I\frac{r^2}{R_1^2} & r<R_1\\ I & R_1<r<R_2\\ 0 & r>R_2\end{cases}$ $B=\begin{cases}\frac{\mu_0 r}{2\pi R_1^2}I & r<R_1\\ \frac{\mu_0\mu_r}{2\pi r}I & R_1<r<R_2\\ 0 & r>R_2\end{cases}$ $\Rightarrow\Phi=\underbrace{\int_0^{R_1}Bdr}_{应为\int_0^{R_1}B\cdot\frac{I'}{I}dr\quad I'=\frac{r^2}{R_1^2}I}+\int_{R_1}^{R_2}Bdr=\frac{\mu_0 I}{4\pi}+\frac{\mu_0\mu_r}{2\pi}I\ln\frac{R_2}{R_1}\Rightarrow L=\frac{\Phi}{I}=\frac{\mu_0\mu_r}{2\pi}\ln\frac{R_2}{R_1}+\underbrace{\frac{\mu_0}{4\pi}}_{\frac{\mu_0}{8\pi}}$ ✗

$\oint\vec{H}d\vec{l}=2\pi rH$

此部分不能直接除I, 是错误的

理想耦合时互感 $M=\sqrt{L_1L_2}$

串联顺接: $L=L_1+L_2+2M\overset{理}{=}(\sqrt{L_1}+\sqrt{L_2})^2$. 反接: $L=L_1+L_2-2M\overset{理}{=}(\sqrt{L_1}-\sqrt{L_2})^2$

并联异名端: $L=\frac{L_1L_2-M^2}{L_1+L_2-2M}$ 同名: $L=\frac{L_1L_2-M^2}{L_1+L_2+2M}$

$\downarrow$ 理 $=0$ $\downarrow$ 理 $=0$

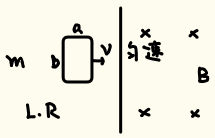
特例: $L_1=L_2=L_0$ 时 理想情况下仍为 L_0.

R-C电路: 充: $q=q_0(1-e^{-\frac{t}{\tau}})$, $\tau=RC$

放: $q=q_0e^{-\frac{t}{\tau}}$.

R-L电路: 充: $I=I_0(1-e^{-\frac{t}{\tau}})$, $\tau=\frac{L}{R}$

放: $I=I_0e^{-\frac{t}{\tau}}$



水平光滑桌面, t=0时开进

$i(t)=?$

$\tau_L=\frac{L}{R}$

$i=i_0'\cdot e^{-\frac{t-t_0}{\tau}}$

不要忘减 t_0 ☆

充电: $\varepsilon=Bbv$

$i=i_0(1-e^{-\frac{t}{\tau}})$

$i_0=\frac{\varepsilon}{R}$

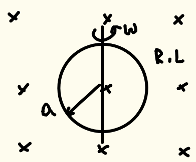
$t_0=\frac{a}{v}$

$(0<t<t_0)$

放电: $\cancel{i=i_0'(1-e^{-\frac{t-t_0}{\tau}})}$

$i_0'=i_0(1-e^{-\frac{t_0}{\tau}})$

$\varepsilon=RI+L\frac{dI}{dt}+\frac{1}{C}\int Idt$



$i(t)=?$

$\begin{cases}\varepsilon=iR+L\frac{di}{dt}\\ \varepsilon=-\frac{d\Phi}{dt}=B\pi a^2w\sin wt\end{cases}$

电流取Re. 阻抗取 | · |

$\tilde{\varepsilon}=\varepsilon_0e^{j(wt+\varphi_e)}$ $\varphi_e=-\frac{\pi}{2}$, $\varepsilon_0=B\pi a^2w$

$\tilde{z}=R+jwL=\sqrt{R^2+w^2L^2}e^{j\varphi}$, $\tan\varphi=\frac{wL}{R}$

$\tilde{I}=\frac{\tilde{\varepsilon}}{\tilde{z}}=\frac{\varepsilon_0}{\sqrt{R^2+w^2L^2}}e^{j(wt+\varphi_e-\varphi)}$

$\Rightarrow i(t)=\mathrm{Re}(\tilde{I})=\frac{\varepsilon_0}{\sqrt{R^2+w^2L^2}}\cos(wt-\frac{\pi}{2}-\varphi)$

磁能密度: $\frac{1}{2}\vec{B}\vec{H}$

交流电路: $\tilde{U}=U_me^{j(wt+\varphi_u)}$ $\tilde{I}$, $\tilde{z}$ 同理. $\tilde{U}=\tilde{I}\tilde{z}$ $\tilde{z}$ 为阻抗, 其中 $\tilde{z}_R=R$, $\tilde{z}_C=\frac{1}{jwC}$ $\tilde{z}_L=jwL$, 复阻抗取模即为阻抗

L上的电压: $L\frac{dI}{dt}$

$t=0$时接通K　$\varepsilon = \frac{Q}{C} + L\frac{d^2Q}{dt^2}$　$\left(\varepsilon = \frac{1}{C}\int I\,dt + L\frac{dI}{dt}\right)$

(1) L中磁场能 = C中电场能的 t_1 (第一次)

(2) L中磁场能第二次达最大值 t_2

解: (1) $\frac{Q}{C} + L\frac{d^2Q}{dt^2} = 0$　$i = \frac{dQ}{dt}$

$Q = Q_0\cos(\omega_0 t + \varphi)$　$\omega_0 = \frac{1}{\sqrt{LC}}$

$t=0$时 $Q=Q_0$. $i=0$ $\Rightarrow Q = Q_0\cos\omega_0 t$

$i = -\omega_0 Q_0 \sin\omega_0 t$

$W_e = \frac{1}{2}\cdot\frac{Q^2}{C} = \frac{1}{2}\frac{Q_0^2}{C}\cos^2(\omega_0 t)$

$W_m = \frac{1}{2}Li^2 = \frac{1}{2}\frac{Q_0^2}{C}\sin^2(\omega_0 t)$

$\Rightarrow t_1 = \frac{\pi}{4}\sqrt{LC}$

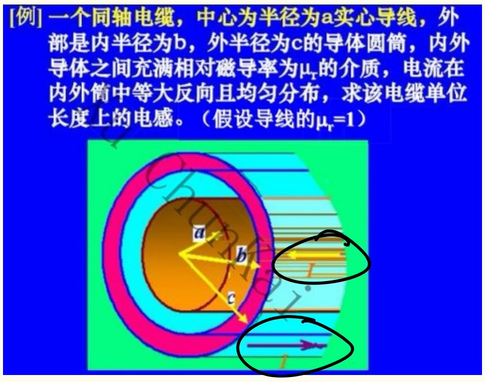
(2) $\omega_0 t = \frac{3\pi}{2} \Rightarrow t = \frac{3\pi}{2}\sqrt{LC}$

两线圈磁能: $W_m = \frac{1}{2}L_1I_1^2 + \frac{1}{2}L_2I_2^2 + MI_1I_2$

$W_m = \frac{1}{2}\sum_{i=1}^{N} I_i\Phi_i$　Φ_i为第i个线圈的全磁通

互感磁能 $W = \Phi_{12}I_2 = M_{12}I_1I_2$　只考虑互感的条件是回路磁能不变(I_1, I_2不变)

对均匀外场 $W_m = I\vec{S}\cdot\vec{B} = \vec{m}\cdot\vec{B}$


[例] 一个同轴电缆，中心为半径为a实心导线，外部是内半径为b，外半径为c的导体圆筒，内外导体之间充满相对磁导率为μ_r的介质，电流在内外筒中等大反向且均匀分布，求该电缆单位长度上的电感。(假设导线的μ_r=1)

$\oint \vec{H}\,d\vec{l} = I_{内}$

$\Rightarrow 2\pi rH = \begin{cases} \frac{r^2}{a^2}I & r<a \\ I & a<r<b \\ I - \frac{r^2-b^2}{c^2-b^2}I = \frac{c^2-r^2}{c^2-b^2}I & b<r<c \\ 0 & r>c \end{cases}$　则 $H = \begin{cases} \frac{r}{2\pi a^2}I & r<a \\ \frac{I}{2\pi r} & a<r<b \\ \frac{c^2-r^2}{c^2-b^2}\cdot\frac{I}{2\pi r} & b<r<c \end{cases}$

① $r<a$

$w_{m_1} = \frac{1}{2}\vec{B}\cdot\vec{H} = \frac{1}{2}\mu_0 H^2 = \frac{\mu_0 r^2}{8\pi^2 a^4}I^2$　$W_{m_1} = \iiint w_{m_1}dV$ 7体积分公式要写!

$= \int_0^a 2\pi r\cdot w_{m_1}dr = \frac{\mu_0 I^2}{16\pi}$

② $a<r<b$

$w_{m_2} = \frac{1}{2}\vec{B}\cdot\vec{H} = \frac{1}{2}\mu_0\mu_r H^2 = \frac{\mu_0\mu_r I^2}{8\pi^2 r^2}$　$W_{m_2} = \cdots = \frac{\mu_0\mu_r}{4\pi}\ln\frac{b}{a}$

③ $b<r<c$

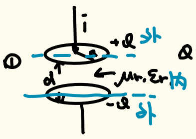
同理有 $W_{m_3} = \frac{\mu_0 I^2}{4\pi(c^2-b^2)^2}\left(c^4\ln\frac{c}{b} - c^2(c^2-b^2) + \frac{1}{4}(c^4-b^4)\right)$

$W = W_{m_1} + W_{m_2} + W_{m_3} = \cdots = \frac{1}{2}LI^2$ 则 $L = \frac{2W}{I^2}$　((结果不用算出)

反直觉: 磁场做功时磁场能增加! 电池做功等于它们的和

计算位移电流密度 $\frac{\partial D}{\partial t}$ $\begin{cases} ① & E_{静} \Leftarrow \oiint \vec{D}\,d\vec{S} = q_0 \\ ② & E_{涡} \Leftarrow \oint \vec{E}\,d\vec{l} = -\frac{d\Phi}{dt} \\ ③ & \vec{j} = \sigma\vec{E} \end{cases}$

①



$Q = Q_0\cos\omega t$　1° 求位移电流密度 j_D

2° 求磁场B的分布　(忽略B变化产生的E)

1° 内 $r<a$. $D = \sigma_0 = \frac{Q}{\pi a^2} = \frac{Q_0}{\pi a^2}\cos\omega t$, $j_D = \frac{\partial D}{\partial t} = -\omega\frac{Q_0}{\pi a^2}\sin\omega t$

其他 $j_D = 0$ ☆

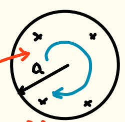
2° 内. $\oint \vec{H}\,d\vec{l} = \iint_S(\vec{j}_0 + \frac{\partial \vec{D}}{\partial t})\,d\vec{S}$　(此时传导电流密度$j_0=0$)

即 $2\pi rH = \begin{cases} -\omega\frac{Q_0}{\pi a^2}\sin\omega t\cdot\pi r^2 & r<a \\ -\omega\frac{Q_0}{\pi a^2}\sin\omega t\cdot\pi a^2 & r>a \end{cases} \Rightarrow H = \begin{cases} \frac{-Q_0\omega r}{2\pi a^2}\sin\omega t & r<a \\ \frac{-Q_0\omega}{2\pi r}\sin\omega t & r>a \end{cases}$　$B = \mu_0\mu_r H = \begin{cases} \mu_0\mu_r H & r<a \\ \mu_0 H & r>a \end{cases}$

外. $\oint \vec{H}\,d\vec{l} = i = \frac{dQ}{dt}$

即 $2\pi rH = -Q_0\omega\sin\omega t$　H可求. B可求.

②



$B = B_0\cos(\omega t)$　(忽略E变化产生的B)

求位移电流密度 j_D

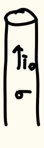
(或螺线管产生.

$I = I_0\cos\omega t$

$B = \mu_0\mu_r nI$)

$\oint \vec{E}\,d\vec{l} = E\cdot 2\pi r = \begin{cases} -\pi r^2\cdot\frac{dB}{dt} & r<a \\ -\pi a^2\cdot\frac{dB}{dt} & r>a \end{cases} = \begin{cases} \frac{r}{2}B_0\omega\sin\omega t & r<a \\ \frac{a^2}{2r}B_0\omega\sin\omega t & r>a \end{cases}$　$D = \varepsilon_0 E \Rightarrow j_D = \frac{\partial D}{\partial t} = \begin{cases} \varepsilon_0\frac{r}{2}B_0\omega^2\cos\omega t & r<a \\ \varepsilon_0\frac{a^2}{2r}B_0\omega^2\cos\omega t & r>a \end{cases}$

③

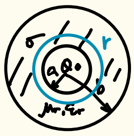


$i_0 = I_0\cos\omega t$

$\begin{cases} \vec{j}_0 = \sigma\vec{E}. \\ \vec{j}_0 = \frac{i_0}{S} = \frac{I_0}{S}\cos\omega t \end{cases} \Rightarrow E = \frac{1}{\sigma}j_0 = \frac{I_0}{\sigma S}\cos\omega t$ 则 $D = \varepsilon_0\varepsilon_r E = \frac{\varepsilon_0\varepsilon_r}{\sigma}\cdot\frac{I_0}{S}\cos\omega t$

$j_D = \frac{\partial D}{\partial t} = -\omega\frac{\varepsilon_0\varepsilon_r}{\sigma}\cdot\frac{I_0}{S}\sin\omega t$

$i_D = j_D\cdot S = -\frac{I_0\varepsilon_0\varepsilon_r\omega}{\sigma}\sin\omega t$



$t=0$ 时内球带电 Q_0

① t 时刻 J_0 ② B

① $\begin{cases} \oiint \vec{D}\, d\vec{s} = q_0 & \Rightarrow DS = q_0 \\ \oiint \vec{J_0}\, d\vec{s} = -\dfrac{dq_0}{dt} & \Rightarrow J_0 S = -\dfrac{dq_0}{dt} = -\dfrac{dD}{dt}\cdot S \\ J_0 = \sigma \vec{E} & \Rightarrow \sigma E = -\varepsilon_0\varepsilon_r \dfrac{dE}{dt} \end{cases}$

则 $E = E_0 \cdot e^{-\frac{\sigma}{\varepsilon_0\varepsilon_r}t}$

$E_0 = \dfrac{Q_0}{4\pi\varepsilon_0\varepsilon_r r^2}$

$\Rightarrow J_0 = \sigma E = \dfrac{\sigma Q_0}{4\pi\varepsilon_0\varepsilon_r r^2} e^{-\frac{\sigma}{\varepsilon_0\varepsilon_r}t}$

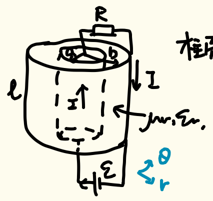
② $J_D = \dfrac{\partial D}{\partial t} = \varepsilon_0\varepsilon_r \dfrac{\partial E}{\partial t} = -\dfrac{Q_0\sigma}{4\pi\varepsilon_r\varepsilon_0 r^2} e^{-\frac{\sigma}{\varepsilon_0\varepsilon_r}t}$

于是 J_t (全电流) $= J_0 + J_D = 0$ 无磁场. $B=0$.

Maxwell 方程组:

$$\begin{cases} \oiint \vec{D}\, d\vec{s} = \sum q_0 \\ \oint \vec{E}\, d\vec{l} = -\iint_S \dfrac{\partial \vec{B}}{\partial t} d\vec{s} \\ \oiint \vec{B}\, d\vec{s} = 0 \\ \oint \vec{H}\, d\vec{l} = \sum I_0 + \iint_S \dfrac{\partial \vec{D}}{\partial t} d\vec{s} \end{cases} \qquad \begin{cases} \nabla\cdot\vec{D} = \rho_0 \\ \nabla\times\vec{E} = -\dfrac{\partial \vec{B}}{\partial t} \\ \nabla\cdot\vec{B} = 0 \\ \nabla\times H = i_0 + \dfrac{\partial D}{\partial t} \end{cases}$$

能流密度 $\vec{S} = \vec{E}\times\vec{H}$

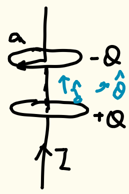


柱壳, $I = \dfrac{\varepsilon}{R}$. 求 $\vec{S} = ?$

有 $E \propto \dfrac{1}{r}$, 设 $E = \dfrac{c}{r} \Rightarrow \varepsilon = \int_a^b \dfrac{c}{r} dr = c\ln\dfrac{b}{a} \Rightarrow c = \dfrac{\varepsilon}{\ln\frac{b}{a}}$ 于是 $E = \dfrac{\varepsilon}{r\ln\frac{b}{a}}\hat{r}$

$\oint \vec{H}\, d\vec{l} = I \Rightarrow \vec{H} = \dfrac{I}{2\pi r}\hat{\theta} \Rightarrow \vec{S} = \vec{E}\times\vec{H} = \dfrac{\varepsilon}{r\ln\frac{b}{a}}\cdot\dfrac{I}{2\pi r}(\hat{r}\times\hat{\theta}) = \dfrac{\varepsilon I}{2\pi r^2\ln\frac{b}{a}}\hat{z}$

$\iint \vec{S}\, d\vec{A} = \int_a^b \dfrac{\varepsilon I}{2\pi r^2\ln\frac{b}{a}}\cdot 2\pi r\, dr = \varepsilon I = I^2 R$. (单位时间流出曲面A的总能量)

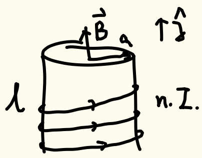


$Q = It$, $E = \dfrac{Q}{\varepsilon_0 S} = \dfrac{It}{\varepsilon_0\pi a^2}\hat{z}$

$D = \varepsilon_0 E \Rightarrow J_D = \dfrac{\partial D}{\partial t} = \dfrac{I}{\pi a^2}$ 于是 $r=a$ 时 $I = I_D$.

$\oint \vec{H}\, d\vec{l} = (\underset{\substack{\| \\ 0}}{I_0} + I_D)_{内} = \begin{cases} r<a & J_D\cdot\pi r^2 = \dfrac{r^2}{a^2}I \\ r\geq a & J_D\cdot\pi a^2 = I \end{cases} \qquad H = \begin{cases} \dfrac{r}{2\pi a^2}I & r<a \\ \dfrac{I}{2\pi r} & r\geq a \end{cases} \hat{\theta}$

于是 $\vec{S} = \vec{E}\times\vec{H} = \begin{cases} \dfrac{I^2 rt}{2\varepsilon_0\pi^2 a^4}(-\hat{r}) \\ 0 \quad (E=0) \end{cases}$ (能量由侧面进入)



$I = kt$ $\vec{S}$?

$\vec{B} = \mu_0 nI\hat{z} = k\mu_0 nt\hat{z}$

$\oint \vec{E}\, d\vec{l} = -\dfrac{d\Phi}{dt} \to 2\pi r\vec{E} = -\dfrac{dB}{dt}\pi r^2 \Rightarrow \vec{E} = -\dfrac{r}{2}k\mu_0 n\hat{\theta}$

$\vec{S} = \vec{E}\times\vec{H} = -\dfrac{r}{2}\mu_0 n^2k^2t\hat{r}$

$\oiint \vec{S}\, d\vec{A} = -\dfrac{a}{2}\mu_0 n^2k^2t\cdot 2\pi a l$ (只有侧面) $= -\pi a^2\mu_0 n^2k^2 t l$

$W_m = \dfrac{B^2}{2\mu_0}\cdot\pi a^2 l \Rightarrow \dfrac{dW_m}{dt} = \dfrac{dW}{dt}$. (能量守恒, 电场能与t无关, 不变)

动量密度 $g = \vec{D}\times\vec{B}$